2-Period Model and Current Account (2)

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GDP and GNI

- Because investment is introduced into the model, the definition of current account shall change. Although all of wealth in the small open economy corresponds to the external assets previously, the wealth corresponds to the sum of the external assets and the capital.
- Thus, the wealth in the end of period t is $B_{t+1}+K_{t+1}$. In this case, the relationship between a change in the wealth and the saving is:

$$B_{t+1}+K_{t+1}-\left(B_t+K_t\right)=Y_t+r_tB_t-C_t-G_t$$

- Plugging this and Eq.(3.12) into Eq.(3.10) yields the definition of the current account when the investment exists in as follows:

$$\begin{aligned} CA_t &= B_{t+1}-B_t \\ &= S_t-I_t \end{aligned} \qquad (3.14)$$

where $S_t \equiv Y_t+rB_t-C_t-G_t$ denotes the national saving. The second line in Eq.(3.14) shows that the difference between the saving and the investment corresponds to the current account.

- The sum of current account and capital account is definitely zero if foreign reserve is negligible. This implies that the surplus in current account definitely corresponds to the deficit in capital account.
- Thus, Eq.(3.14) implies that an increase in the external assets corresponds to the deficit in capital account.

- Now, the concept of current account in the model is consistent with the actual concept because of investment and we mention the relationship between national income account and current account.
- The definition of Gross Domestic Product is given by:

$$Y_t \equiv C_t+I_t+G_t+NX_t \qquad (3.15)$$

Where $NX_t \equiv EX_t-IM_t$ denotes the net export and EX_t and IM_t denote the export and the import, respectively.

- By combining Eqs.(3.14) and (3.15) and the definition of the national saving yields:

$$CA_t = NX_t+rB_t$$

This equality shows that the current account corresponds to the sum of the net export and the interest income.

- GNI is given by Y_t+rB_t Thus, combining this and Eq.(3.15) yields:

$$Y_t+rB_t = C_t+I_t+G_t+CA_t$$

where C_t+I_t+G_t is absorption.

- By comparing this with Eq.(3.15), we can understand that GDP is the sum of absorption and net export while GNI is the sum of absorption and current account.

Investment and Households' Maximization Problem

- Introducing investment into the model changes households' budget constraint. Thus, we reconsider households' maximization problem.
- Paying attention to B_1=0, Eq.(3.14) can be rewritten as:

$$B_2 = Y_1 - C_1 - G_1 - I_1$$

- Similarly, because of B_3=0, Eq.(3.14) also yields:

$$-B_2 = Y_2 + rB_2 - C_2 - G_2 - I_2$$

- By combining these 2 equalities, we have households' intertemporal budget constraint as follows:

$$C_1 + I_1 + \frac{C_2 + I_2}{1+r} = Y_1 - G_1 + \frac{Y_2 - G_2}{1+r} \qquad (3.16)$$

- Households maximize Eq.(3.1) subject to Eq.(3.16). While the budget constraint contains the government expenditure and the investment, still the FONC Eq.(3.3) and Euler equation Eq.(3.4) are applied because households just control the consumption.

Production Possibility Frontier and Equilibrium

- Now, we focus not only indifferent curves and budget constraints but also production possibility frontier under an economy without international indebtedness.
- Production possibility frontier is combinations of feasible and efficient amount of production
- The Production possibility frontier without international indebtedness is given by:

$$C_2 = \underbrace{F\left[K_1 + Y_1 - C_1\right]}_{Y_2} + \underbrace{K_1 + \underbrace{Y_1 - C_1}_{I_1}}_{K_2} \qquad (3.17)$$

- Let assume that this economy choose minimum investment in period 1 and consumes all of accumulated capital, that is, the economy chooses $I_1 = -K_1$ ($I_1 + K_1 = 0$).
- In this case, the consumption period 1 is $C_1 = K_1 + F(K_1)$, which is extremely high ($C_1 = K_1 + Y_1$).
- However, because of zero capital accumulation, the consumption in period 2 is $C_2 = 0$.

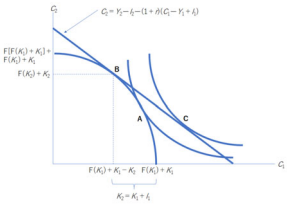
- On contrary, households do not consume the capital in the end of period zero K_1 and all of the output in period 1 assign to investment ($I_1 = Y_1$).
- In this case, the consumption and the investment in period 1 are $C_1 = 0$ and $I_1 = F(K_1)$, respectively.
- In the period 2, those are $C_2 = F\left[K_1 + F(K_1)\right] + K_1 + F(K_1)$ and $K_2 = K_1 + F(K_1)$, respectively ($C_2 = F(K_1 + Y_1) + K_1 + Y_1$, $K_2 = K_1 + Y_1$).
- Although the consumption in period 1 is zero, it in the period 2 is extremely high, as shown in the intercept, in this case (Fig. 3-2).

- The slop of the PPF is derived from Eq.(3.17) and is given by:

$$\frac{\partial C_2}{\partial C_1} = -\left[1 + F'(K_2)\right]$$

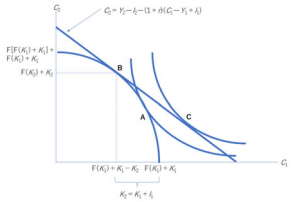
- This implies that the PPF is strictly concave because of diminishing marginal productivity.

Fig. 3-2: the PPF and the Consumption determined



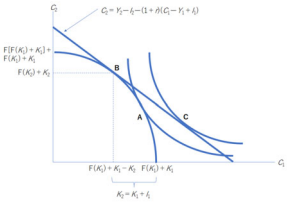
- Fig. 3-2 shows the relationship between the consumption in 2 periods and the current account if there is the investment instead of Fig. 3.1.
- The PPF shows the combination of the highest consumption when there is not international indebtedness.

Fig. 3-2: the FFP and the Consumtion determined



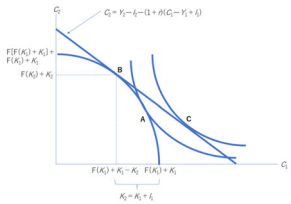
- In point A, the slop of both the indifferent curve and the FFP is $-(1+r^A)$ where r^Adenotes the real interest rate in a closed economy.
- On one hand, the slope of the budget constraint is given by $-(1+r)$. This implies $r^A > r$.
- In this economy where is international indebtedness is given by r which is the world interest rate.

Fig. 3-2: the FFP and the Consumtion determined



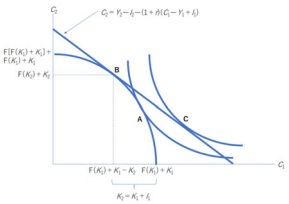
- Thus, the earning rate on domestic investment exceeds the world interest rate in point A.

Fig. 3-2: the FFP and the Consumtion determined



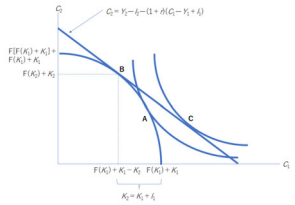
- The higher return rate attracts foreign investors and they start to invest in the home. The production increase and the output goes to point B where the budget constraint crosses with the FFP.
- Similar to Fig. 3-1 analyzing consumption under the model without investment, the consumption is point C where the indifferent curve crosses with the budget constraint.

Fig. 3-2: the FFP and the Consumtion determined



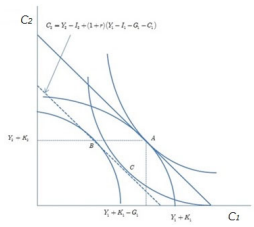
- Fig. 3-2 shows that output is not point A but point B.
- That is, the investment from the foreign enhances households' utility.

Fig: 3-3: The Government Expenditure and the Current Account



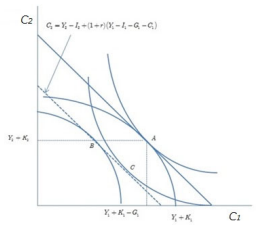
- Now, we introduce not only the investment but also the government expenditure.
- The budget constraint Eq.(3.16) and the PPF Eq.(3.17) explain how a change in the government expenditure affects the consumption.

Fig: 3-3: The Government Expenditure and the Current Account



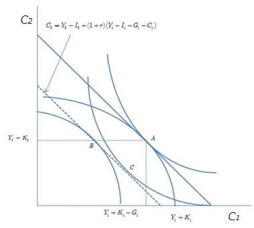
- An increase in the government expenditure in period 1 shifts Eqs.(3.16) and (3.17) left side by the same increase.
- An increase in the government expenditure in period 2 shifts Eqs.(3.16) and (3.17) downward by the same increase.

Fig: 3-3: The Government Expenditure and the Current Account



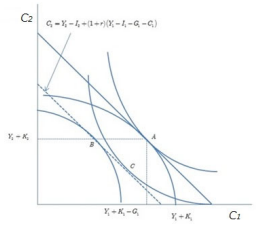
- Point A in Fig. 3-3 shows the consumption when both the government expenditure and the current account are zero.
- This can be understood easily because point A is an intersection of the indifferent curve and the PPF.

Fig: 3-3: The Government Expenditure and the Current Account



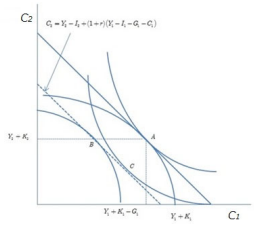
- Let assume $G_1 > T_1$ and $G_2 = 0$. In this case, both the budget constraint and the PPF shift left side by G_1 and the production is point B.
- Because of international indebtedness, the consumption is point C where both shifted budget constraint and the Indifferent Curve cross.

Fig: 3-3: The Government Expenditure and the Current Account



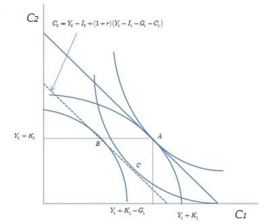
- Thus, An increase in the government expenditure in period 1 induces the current account deficit in period 1.

Fig: 3-3: The Government Expenditure and the Current Account



- An increase in the government expenditure in period 2 shifts both the budget constraint and the FFP downward and induces the current account surplus in period 1.
- While an increase in the government expenditure decreases the disposable income, households make the consumption constant over time.

Fig: 3-3: The Government Expenditure and the Current Account



- Thus, the current account deficits or surplus occur through international indebtedness.

Tab. 3-1: Fiscal Balance and Current Account

		Fiscal Balance	
		Period 1	Period 2
Current Account	Period 1	Deficit	Surplus
	Period 2	Surplus	Deficit

- Tab. 3-1 shows the relationship between the government expenditure and the current account.